# Estimation of Soil and Crop Hydraulic Properties

Theodor S. Strelkoff, M.ASCE[1]; Albert J. Clemmens, M.ASCE[2]; and Eduardo Bautista, M.ASCE[3]

**Abstract:** Some two dozen methods for estimating infiltration and roughness parameters from field measurements of test irrigations are reviewed in this paper. They differ in their assumptions, ease of analysis, quantity of field data required, and accuracy. They are divided into two broad categories, depending upon the basic approach to determine infiltration. One features direct application of mass conservation, expressed in terms of the infiltration parameters and then inverted in some way in order to extract those parameters. The other involves repeated simulation with a sequence of values of the infiltration parameters, coupled to some kind of search procedure—an optimization—to minimize differences between simulation and measurement. A new one-point technique is proposed, along with suggestions for extending existing methods.

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## Introduction

Effective evaluation, management, design, and simulation of surface-irrigation systems depend on the knowledge of the hydraulic characteristics of the fields over which the irrigating streams flow. A brief review of general considerations regarding the field parameters influencing surface irrigation can be found in the companion paper, Strelkoff et al. (2009), and the present exposition can be thought of as a continuation of that document, in particular, as regards infiltration, soil-surface roughness, and crop hydraulic drag. The last two are usually lumped together and identified simply as roughness. Infiltration and roughness parameters are generally expressed through coefficients in some sort of functional form.

Unlike soil-surface elevations, also important in performance, these field characteristics cannot, with current instrumentation and theory, be directly measured prior to an event. Current trends are to measure data during an irrigation event and deduce average or effective infiltration and roughness from these measurements. Some two dozen methods have been proposed in the literature, varying widely in data requirements, sought-after parameters, assumptions, complexity of analysis, and resultant accuracy. A few methods allow some degree of determination of spatial variability.

The Environmental and Water Resources Institute of ASCE (EWRI/ASCE) Task Committee on Soil and Crop Hydraulic Properties has undertaken to formulate an annotated review of field evaluation methods. As a step in achieving that goal, this paper provides brief descriptions of various methods, mostly available in existing literature, emphasizing their shared approaches and individual differences. Infiltration, generally regarded as difficult to estimate, in part because of spatial and temporal variations, and yet extremely important to irrigation performance, is stressed in the paper, followed by roughness considerations. The important influence and estimation of spatial variability is not emphasized, nor are real-time determinations of field properties, which can play a role in controlling an ongoing irrigation.

From the theoretical standpoint, there are two fundamental approaches for estimating infiltration. One comprises a more-or-less direct computation, based on a volume balance applicable over any time interval. The volume of inflow must balance the volume of the surface stream, the infiltrated volume, and the outflow volume, for any segment of an irrigated length, as well as for the entire length. Depending upon the assumptions made, an inversion of the equations governing the surface-irrigation stream flow can sometimes be made, yielding a direct, or essentially direct (e.g., a regression analysis) solution. The other approach rests upon a match between measured irrigation data and computer simulations of the irrigation performed with a sequence of values for the sought-after infiltration (and roughness) parameters. The simulations are repeated in some kind of formal optimization procedure involving a search for the unknown field parameters. This paper reviews methods in the literature that fall into one or the other category.

In the 1980's and 1990's, in response to an interest in controlling surface irrigations with the aid of real-time measurements (typically advance), a number of approaches appeared in the literature, many of which adjusted estimated infiltration parameters at a sequence of time levels by minimizing the differences between theoretical simulations and measured advance. These methods were found, by and large, to do a much better, more robust job of estimating infiltration parameters if the entire advance was taken into account, rather than individual increments of advance, step by step, or partial advance. This is not surprising since irregularities in measured advance rates, stemming from longitudi-

[1]Research Hydraulic Engineer, U.S. Arid Land Agricultural Research Center, USDA-ARS, 21881 N. Cardon Ln., Maricopa, AZ 85238 (corresponding author). E-mail: Theodor.Strelkoff@ars.usda.gov

[2]Research Hydraulic Engineer, Center Director, U.S. Arid Land Agricultural Research Center, USDA-ARS, 21881 N. Cardon Ln., Maricopa, AZ 85238. E-mail: Bert.Clemmens@ars.usda.gov

[3]Research Hydraulic Engineer, U.S. Arid Land Agricultural Research Center, USDA-ARS, 21881 N. Cardon Ln., Maricopa, AZ 85238. E-mail: Eduardo.Bautista@ars.usda.gov

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nal spatial variations in cross section, bottom configuration, and roughness as found in the field, were expected to be accounted for theoretically solely by infiltration-rate variations. This paper confines its attention to parameter-estimation methods based on the entire measured irrigation, rather than real-time determinations made during the advance process.

The paper is organized as follows: empirical formulas for infiltration and roughness in common use are presented [see also Strelkoff et al. (2009)], the first as a function of wetting time, and the other as a function of slope, wetted cross section, and discharge rate. The necessary balance of volumes is reviewed next, followed by a series of methods that make direct use of the mass conservation principle. The application of Bayesian statistics to generate estimates for missing parameters is noted. The Lewis and Milne equation, incorporating a particularly surface irrigation-oriented expression for infiltrated volume is introduced followed by a method for extracting the parameters of the infiltration formula from the balance. The far-reaching simplifications that arise when the stream-advance function of time is assumed to be well fitted by a power law are noted next, for both the advance and postadvance periods. Methods based on this assumption follow. The common assumptions for estimating surface-water volumes during the irrigation, in lieu of comprehensive surface depth measurements are introduced, followed by the methods incorporating this very significant assumption. Techniques involving surface-irrigation simulations matching measured stream behavior, short of formal multidimensional optimization, are presented, as well as several formal optimization methods.

## Mathematical Expression of Field Characteristics

Cumulative infiltration is usually expressed as $z$, volume per unit area, in border strips and basins, and $A_z$ (sometimes given the symbol $Z$ in the literature), intake volume per unit length, in furrows. Both are generally recognized as dependent on infiltration time $\tau$. Furrow intake is increasingly recognized as often dependent on wetted perimeter as well. The hydrostatic pressure on the water flowing over or ponded on the soil surface also plays a small role, amounting to about 2% increase per centimeter of surface-water depth (Philip 1958,1969). The empirical functional form most common in surface irrigation studies for expressing the dependence of $z$ on time is a modified Kostiakov formula

$$z = k\tau^a + b\tau + c \tag{1}$$

with $k$, $a$, $b$, and $c$ empirical constants ($b$ and especially $c$ are often set to zero). The Kostiakov-Clemmens branch function (Kostiakov 1932; Clemmens 1981), which reaches the basic rate sooner, is given (with the addition of the constant term $c$) by the two branches joined at $\tau_B$

$$z = c + k\tau^a, \quad \tau < \tau_B; \quad z = c_B + b\tau, \quad \tau > \tau_B \tag{2a}$$

in which

$$\tau_B = \left(\frac{ak}{b}\right)^{1/1-a}; \quad c_B = c + k\tau_B^a - b\tau_B \tag{2b}$$

The same forms as Eqs. (1) and (2) have been used for intake, $A_z$, though the values of the constants (and their units) will change radically. We will use capital letters ($K$, etc.) to signify the parameters in the corresponding intake formula. The quantities $z$ and $A_z$ are related through the wetted perimeter or other characteristic width, $W$; as a very rough concept, $A_Z = Wz$ (clearly, as wetted perimeter falls after cutoff, $A_Z$ continues to increase, albeit more slowly). In particular, if furrow infiltration is assumed based on furrow spacing one unit wide, $K=k$, $B=b$, etc. Caveats regarding the role of wetted perimeter are noted in Strelkoff et al. (2009). Of particular importance, with a given soil $k$, calculated intake per unit length in a furrow can vary a great deal depending on whether local wetted perimeter, wetted perimeter at the upstream end, wetted perimeter at normal depth, or furrow spacing is used as a basis. The NRCS in applying their families, for example, relate intake per unit length to infiltration per unit infiltrating area through an empirical wetted perimeter, related to normal depth at the upstream end. It follows, in general, that a given measurable $K$ will relate differently to $k$ values derived with different wetted perimeter assumptions.

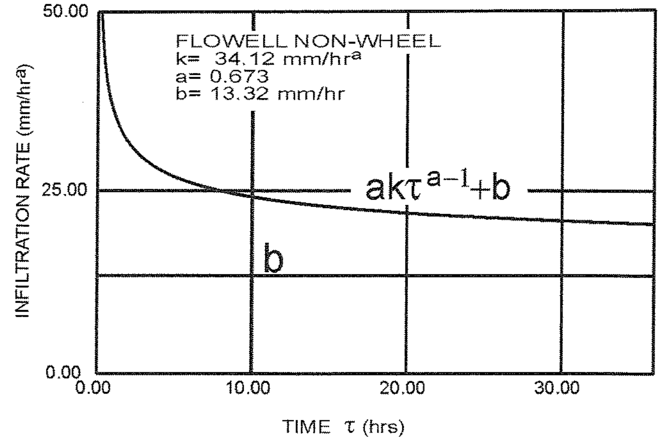


**Fig. 1.** Infiltration rate contrasted with final basic rate. Modified Kostiakov formula. Field depth of infiltration based on 1-m furrow spacing.

It is often assumed that the basic rate of infiltration is achieved by the end of the irrigation and can be found by simultaneously measuring the rates of outflow $Q_{RO}$ and inflow $Q_0$, i.e.

$$b = \frac{Q_0 - Q_{RO}}{LW} \tag{3}$$

in which $L$=furrow length and $W$=representative wetted perimeter—use of the furrow spacing here would make $z$ a field depth. It should be noted, however, the contribution of the Kostiakov power term in Eq. (1) to infiltration rate can be considerable, even after many hours. See, e.g., Fig. 1, drawn for the Flowell nonwheel furrow conditions (free of compression by tractor wheels) cited in Walker and Humpherys (1983). After 8 h, the power term still contributes 47% of the total infiltration rate. Even after 24 h, the contribution has only dropped to 37%.

The selection of one or another functional form for data derived solely from a test irrigation in a border strip or furrow is not as straightforward as with ring-infiltrometer data, but, still, an a priori choice provides stability in the procedure for estimating the parameters for that form.

Soil-surface roughness and plant drag on the flowing stream are commonly expressed through the Manning formula [see Strelkoff et al. (2009) for a brief discussion of the issues]

$$S_f = \frac{Q^2 n^2}{A_Y^2 R^{4/3}} \tag{4}$$

in which $Q$=flow rate; $A_Y$=cross-sectional area of flow; $R$ =hydraulic radius (area divided by wetted perimeter); $S_f$ =friction slope, essentially, the slope of the water surface; and

$n$=empirical Manning roughness parameter characterizing the soil grains, clods, plant parts, etc.

## Mass Conservation—Volume Balance

Explicit expression of mass conservation forms the basis for all of the direct methods of parameter estimation. During a surface irrigation the mass balance can be written

$$V_Q(t) = V_Y(t) + V_Z(t) + V_{RO}(t) \tag{5}$$

in which inflow volume $V_Q$, the surface and infiltrated volumes, $V_Y$ and $V_Z$, respectively, and the volume of runoff $V_{RO}$ are in balance at every instant of time $t$. These can be expressed in terms of rates, as follows, for example, prior to cutoff

$$V_Q = \overline{\overline{Q}}_0(t) \cdot t \tag{6}$$

in which $\overline{\overline{Q}}_0$ is the time-averaged inflow rate, and

$$V_Y = \overline{A_Y} \cdot x_A \tag{7}$$

with $\overline{A_Y}$=distance-averaged cross-sectional area of the surface stream over the advance distance, $x_A$. The runoff volume is

$$V_{RO} = \int_0^t Q_{RO} dt \tag{8}$$

the integral form underscoring that runoff is extremely variable. Runoff is zero during all of the advance, then typically rises quickly to a shoulder, continuing to rise gradually as infiltration rates decrease with time, and finally returns back to zero more-or-less gradually after cutoff.

The infiltrated volume is given by

$$V_Z(t) = \int_0^{x_A(t)} A_Z(x,t) dx \tag{9}$$

in which $A_Z$=infiltrated volume per unit length. If infiltration is assumed a function only of wetting time [ignoring hydrostatic pressure or wetted perimeter effects, e.g., Eq. (1) and (2)], Eq. (9) can be written

$$V_Z(t) = \int_0^{x_A(t)} A_Z(t - t_A[x]) dx \tag{10}$$

which contains both the advance function [$x_A(t)$ or $t_A(x)$, which can be measured] and the infiltration function (sought). Many of the direct parameter-estimation methods are based on a known $V_Z(t)$, with inflow and outflow in Eq. (5) measured, and $\overline{A_Y}$ from either measured surface-water depths or an estimate.

### Estimation on a Field with a Prior History of Parameter Estimates

While many of the simple methods take advantage of some knowledge of the conditions of the field, or perhaps use a parameter determined from prior irrigation events, in general, these techniques treat parameter estimation as if the collected data constituted the first observations stemming from the unknown parameters. Clemmens and Keats (1992a,b) used Bayesian statistics to identify and take into account simulation-model bias (the result of differences between model assumptions and the real world). Statistical analysis of differences between model output and measured values yields learned patterns of bias that can be applied to current data to increase its predictive capabilities. Furthermore, the approach allows statistical incorporation of parameter values determined in previous tests into the process to increase the accuracy of current (especially, real-time) estimates. The main thrust, however, has been to use prior history to estimate two parameters when measured data and standard estimation methods yield only one.

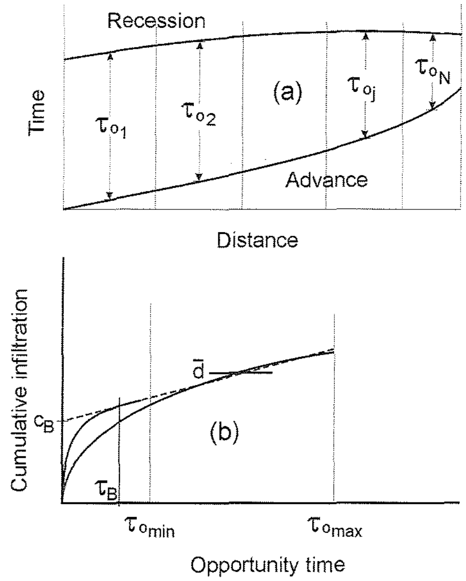


**Fig. 2.** (a) Postirrigation distribution of opportunity times; (b) different infiltration functions satisfying the postirrigation infiltration distribution

### Postirrigation Determination of Infiltration Characteristics

A particularly simple and theoretically exact application of the volume balance to evaluation of the infiltration parameters stems from the postirrigation analysis of measured advance and recession curves. With the irrigation completed, all surface water has either drained off or infiltrated, $V_Y$ in Eq. (5) is zero, and $V_Z$ is known from the measured inflow and outflow. Thus the average depth of infiltration is known. Indeed, following an initial suggestion by Merriam (1971), Clemmens (1981) developed equations for whole-border estimates of Kostiakov $k$, if the exponent $a$ is known from other sources. If the basic Kostiakov formula, $k\tau_o^a$, is assumed, in which $\tau_o(x)$ is the infiltration opportunity time between the advance and recession curves, as in Fig. 2, and if a is taken, for example from ring data, $k$ can be found from the equation for the total volume infiltrated per unit width

$$V_Z = kW \sum_{j=1}^{N} \tau_{oj}^a \delta x_j \tag{11}$$

in which the border length has been subdivided into $N$ segments and $\tau_{oj}$=average opportunity time for the $j$th segment; $W$ =effective wetted perimeter, assumed constant, so that $A_Z = W_Z$.

Clemmens (1981) extended the technique of Eq. (11) to the branch function of Eq. (2) for the common case that the basic infiltration rate has been achieved within the smallest opportunity time measured. For example in a border strip, with the postirrigation depth of infiltration $d$ at any station dependent solely on the opportunity time there

$$d = k\tau_B^a + b(\tau_o - \tau_B) \quad (12)$$

the average depth of infiltration over the border (known from measurement of inflow and outflow) is

$$\bar{d} = k\tau_B^a + b(\overline{\tau_o} - \tau_B) \quad (13)$$

With $a$ and $b$ estimated from ring data, the crossover point $\tau_B$ is found in terms of $k$ from the third of Eq. (2), and the result substituted into Eq. (13), to yield the following (simplified, corrected) whole-border estimate of $k$

$$k = \left(\frac{b}{a}\right)^a \left(\frac{\bar{d} - b\overline{\tau_o}}{1-a}\right)^{1-a} \quad (14)$$

If $\tau_B$ is better estimated from the ring data than $b$, $b$ will follow from

$$b = \frac{a\bar{d}}{(1-a)\tau_B + a\overline{\tau_o}} \quad (15)$$

obtained after eliminating $k$ from Eq. (13) and the third of Eq. (2), which then yields $k$.

Fig. 2 illustrates that the details of the infiltration function at opportunity times less than the observed minimum play a minor role in the distribution of infiltrated water over the given length of run, as long as the behavior of the function within the observed range of opportunity times is reasonably correct. The more nearly uniform are the opportunity times distributed over the length of run, the more exact will be the evaluation of the irrigation. At the same time, Clemmens et al. (2001) caution that a narrow range of opportunity times in the parameter estimation can lead to large errors in predicted performance for the same soil but under different hydraulic conditions (e.g., slope, inflow rate, length, etc.); ultimate stream behavior is dependent on the entire infiltration function of time—right up to the end of recession.

It should also be possible to avoid independent assessment of $a$, if two borders with the same soil conditions are irrigated, and their variations in opportunity time are analyzed. In principle, both $k$ and $a$ can be solved by a Newton-Raphson solution of the nonlinear equation pair

$$G_1 = V_{Z1} - kW\sum_{j=1}^{N_1} \tau_{o1j}^a \delta x_{1j} = 0$$

$$G_2 = V_{Z2} - kW\sum_{j=1}^{N_2} \tau_{o2j}^a \delta x_{2j} = 0 \quad (16)$$

for which

$$\frac{\partial G_1}{\partial k} = -W\sum_{j=1}^{N_1} \tau_{o1j}^a \delta x_{1j}$$

$$\frac{\partial G_1}{\partial a} = -kW\sum_{j=1}^{N_1} \tau_{o1j}^a \ln \tau_{o1j} \delta x_{1j}$$

$$\frac{\partial G_2}{\partial k} = -W\sum_{j=1}^{N_2} \tau_{o2j}^a \delta x_{2j}$$

$$\frac{\partial G_2}{\partial a} = -kW\sum_{j=1}^{N_2} \tau_{o2j}^a \ln \tau_{o2j} \delta x_{2j} \quad (17)$$

Overconditioning the problem with more irrigations and equations would allow for a least-squares best fit for both $k$ and $a$ [this multiple-border postirrigation approach is based on that of Bouwer (1957), which in the absence of a functional form for infiltration, leads to a table of cumulative infiltration versus time—that may, like the results of Finkel and Nir (1960) described later, tend to oscillate].

Independent determination of $a$ can also be avoided with soils that can be characterized by membership in empirically defined families (Strelkoff et al. 2009), e.g., the Soil Conservation Service (SCS) [NRCS/USDA (SCS) 1984] families or the Merriam and Clemmens (1985) time-rated families. Each NRCS family member is characterized by particular values of Kostiakov $k$ and $a$, while the time-rated families are characterized by an empirical relationship between $a$ and $t_{100}$, the time to infiltrate 100 mm. The net effect in either case is an implied relationship between $k$ and $a$, given, for the time-rated families, by Eq. (73) in Appendix II. Then, a Newton-Raphson solution for $k$ and $a$ would utilize the first two partial derivatives in Eq. (17), and two more stemming from a restatement of Eq. (73)

$$G_2 = k - 10^{2-a[(0.675-a)/0.2125]} = 0$$

$$\frac{\partial G_2}{\partial k} = 1$$

$$\frac{\partial G_2}{\partial a} = -\ln 10 \cdot 10^{2-a[(0.675-a)/0.2125]} \frac{2a - 0.675}{0.212\,5} \quad (18)$$

Similar equations can be derived for the NRCS infiltration families by using the relationship between $k$ and a depicted in Fig. 8 of Strelkoff et al. (2009) (shown also to a smaller scale in Fig. 10(b), Appendix II) by the fitted function (Valiantzas et al. 2001)

$$k(a) = 60^a \left[14{,}088\ a^{45} + \frac{0.148}{(-\ln a)^{1.652}}\right] \quad (19)$$

### *Parameter Estimation Requiring a Closely Spaced Set of Measured Surface-Depth Hydrographs*

At the other end of the spectrum of required field data, the measurement of a complete set of depth hydrographs along the length of run during the irrigation yields the time variation of the volume temporarily stored on the surface, $V_Y(t)$ in Eq. (5). Thus, with inflow and outflow known, $V_Z(t)$ is also known. Together with the advance and recession curves, $x_A(t)$, $x_R(t)$ (a by-product of the depth hydrographs), this information can be used to deduce the field infiltration parameters.

This class of methods requires the most extensive data collection, but is encumbered by the fewest assumptions, and so is the most direct and physically based of all the techniques reflecting conditions during the entire irrigation. Based almost exclusively on mass conservation, the principle problem lies in extracting the infiltration parameters (and possibly roughness) from the measured data. The techniques, all of which assume that $V_Y(t)$ is a known, measured function of time, differ in the assumptions made on the functional forms of the advance and infiltration functions.

In an early development, Finkel and Nir (1960), making no assumptions on infiltration- or advance-function form, proposed a graphical inversion of the technique by Hall (1956) for advance in border strips, a recursive algebraic equation for the increments of advance at a sequence of advance times, separated by constant $\Delta t$. In principle, a tabulated infiltration function of time could be constructed step by step, a $\Delta z$ for each $\Delta t$, without regard for any functional form of either advance or infiltration function. However, unless great care is taken that the increments in surface volume are accurate, and the construction is limited to a small number of $\Delta t$, the results for $\Delta z$ begin to oscillate. This instability evidently stems from the structure of the governing algebraic equation, the inversion of Hall's. Any error in measurement or calculation (round off, etc.) must be absorbed by the current calculated $\Delta z$, and successive steps in the calculation magnify it; the forward calculation, advance, is stable with the Hall technique, the reverse, for infiltration, is not.

In a direct, interactive computer procedure (EVALUE), Strelkoff et al. (1999) superimposed screen plots of measured $V_Z(t)$ and $V_Z(t)$ calculated from estimates of infiltration parameters in a selected functional form. Variable wetted perimeter is not considered, and the unknown parameters are assumed constant over the entire length of the run. Numerical integration for the volume under the infiltrated profile is enhanced by weighting factors based on the ratio of calculated infiltrated depths at each end of a segment and on an assumed power law (with exponent $a$) for depth versus distance back from the advancing front. The parameter values are changed at the keyboard until the user is satisfied with the fit of the two curves. In addition to the function of time, the final values of infiltrated volume are also plotted for matching. Various functional forms can be investigated simultaneously with the parameters in those forms. This brute-force approach is aimed at selecting those parameters best representative of the entire time span of the irrigation. A by-product of the calculations is a determination of Manning $n$ values at the stations, determined from Eq. (4) by calculating the local discharge from continuity and applying the water-surface slope given by smoothed profiles derived from the measured hydrographs. A representative value for the irrigation is obtained by averaging the results.

Maheshwari et al. (1988), working with irrigation borders made in extensively cracked clay soils ($k$ and $a$ of Eq. (1) were essentially zero in many cases), automated a similar procedure by formally minimizing an objective function, $Z^*$

$$Z^* = \sum_{i=1}^{N} (V_{\mathrm{O}i} - V_{\mathrm{C}i})^2 \tag{20}$$

with the Hooke and Jeeves pattern-search technique attributed to Leon (1966) and Monro (1971). $V_{\mathrm{O}i}$ is an observed infiltrated volume, based on Eq. (5) with measured inflow and outflow rates and $V_Y$ given by numerically integrated measured surface-water depths. $V_{\mathrm{C}i}$ is calculated from current values of infiltration parameters and the measured advance curve. $N$ is the number of times (separated by equal increments) that the comparison is made in the duration of the study, ending typically when recession begins at the upstream end. In application, the measured advance and infiltrated-volume functions of time are fitted with mathematical expressions by regression. A number of different functional forms for infiltration and advance were investigated.

In 1997 (Esfandiari and Maheshwari 1997a), the method was extended to furrows. The modified Kostiakov parameters $K$, $a$, and $B$ were sought, directly yielding intake (volume per unit length). Consequently no explicit consideration of wetted perimeter in infiltration was undertaken.

Other techniques have been designed to extract the infiltration parameters by one or another manipulation of the integral in Eq. (10). The best known of these is the Lewis and Milne equation.

### *Lewis and Milne Integral Equation*

Lewis and Milne (1938) substituted a time integral for the distance integral of Eq. (10) by means of a formal change in variable $dx = dx/dt_x \; dt_x$, in which the derivative is the rate of advance $dx_A/dt(x)$ at the time the stream has advanced to point $x$. Eq. (5) then takes the form

$$\overline{\overline{Q}}_0 \cdot t = \overline{A}_y x + \int_0^{t_U} A_Z(t - t_x)\frac{dx}{dt_x}dt_x + V_{\mathrm{RO}}(t) \tag{21}$$

in which $t_x$=time to advance to $x$. The upper limit on the integral is

$$t_U = t, \quad \text{during advance}$$

$$t_U = t_L, \quad \text{after advance} \tag{22}$$

in which $t_L$=time to advance to the end of the run, $x=L$. The equation is used with a variety of assumptions or measured data for a variety of purposes, both irrigation simulation and field-parameter estimation. Its generality allows it to be applied to various phases of an irrigation. During the advance phase, for example, $V_{\mathrm{RO}}$ is zero, and the rest can be viewed as an integral equation for advance $x_A$ as a function of irrigation time $t$ (e.g., Lewis and Milne 1938; Hall 1956).

Some well known applications of Eq. (21) follow a simplifying assumption on the functional form of the advance curve. These will be described in a subsequent section. A technique free of arbitrary assumptions on the advance function is the linear station advance procedure of Clemmens (1982). Numerical integration of the infiltrated depth profile is enhanced by the change of variables in Eq. (10) yielding

$$\Delta V_{Zj}(t_i) = \int_{t_{Aj-1}}^{t_{Aj}} A_Z(t_i - t_A)\frac{dx}{dt_A}dt_A \tag{23}$$

Now, following Clemmens, who proposed assuming a (different) constant rate of advance over each increment $\Delta x_A$, the advance rate in Eq. (23) can be taken outside the integral and expressed as a quotient of differences, as follows:

$$\Delta V_{Zj}(t_i) = \frac{\Delta x_j}{\Delta t_{Aj}}\int_{t_{Aj-1}}^{t_{Aj}} A_Z(t_i - t_A)dt_A \tag{24}$$

in which $\Delta t_{Aj} = t_A(x_j) - t_A(x_{j-1})$. Now, with any given functional form for $z(\tau)$, the integral in Eq. (24) can be evaluated for each $j = 1, \ldots, N$, and the results summed for a calculated $V_z$ in terms of the infiltration-function parameters, namely

$$V_Z(t_i) = \sum_{j=1}^{N} \Delta V_{Zj}(t_i) \tag{25}$$

For example, with the modified Kostiakov formula of Eq. (1), Eq. (24) appears as

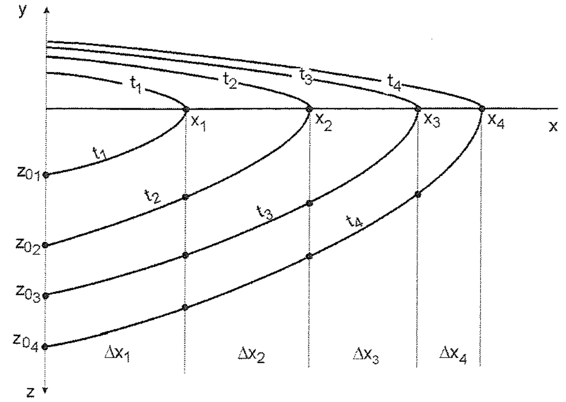



**Fig. 3.** Surface and infiltrated profiles at an arbitrary sequence of time steps

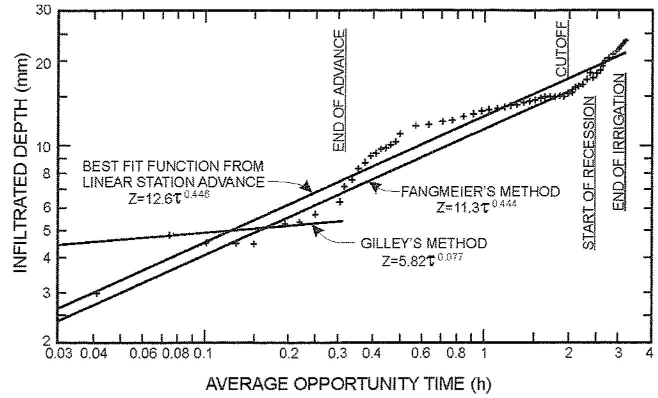



**Fig. 4.** Infiltration functions computed by Linear Station Advance, Fangmeier's method, and the Gilley technique [after Clemmens (1982)]

$$\Delta V_{Zj}(t_i) = \frac{\Delta x_j}{\Delta t_{Aj}}\left[k\left(\frac{\{t_i - t_{Aj-1}\}^{a+1} - \{t_i - t_{Aj}\}^{a+1}}{a+1}\right) + b\left(\frac{\{t_i - t_{Aj-1}\}^2 - \{t_i - t_{Aj}\}^2}{2}\right)\right] + c\Delta x_j \qquad (26)$$

as can be followed in Fig. 3 showing surface and subsurface profiles at a sequence of time steps arbitrary increments of time apart. The first term of Eq. (26) is identical to the result presented by Clemmens.

With the branch function [Eq. (2)], if $t-t_A<\tau_B$, the first and third terms of Eq. (26) are appropriate. For $t-t_A>\tau_B$

$$\Delta V_{Zj} = b\frac{\Delta x_j}{\Delta t_{Aj}}\left(\frac{\{t - t_{Aj-1}\}^2 - \{t - t_{Aj}\}^2}{2}\right) + c_B\Delta x_j \qquad (27)$$

in which $c_B$ is defined in Eq. (2).

The form of Eqs. (26) and (27) allows for spatial variability in the infiltration parameters, but, typically, the equations would be used to estimate whole-border values. With two parameters, $k$ and $a$ in a power law, a minimum of two time levels is required for solution. For the four parameters in the modified Kostiakov formulation of Eq. (26), it would follow that four simultaneous equations of the type of Eq. (25), at each of four time levels, would be the minimum. Typically, many more are used, with a best fit sought.

In seeking global values of Kostiakov $k$ and $a$ for a border irrigation, Clemmens developed provisional values for all $\Sigma_{j=1}^{i}(j-1)$ possible $k$ and $a$ pairs derived from all the time steps leading to a given time level $i$ (for example, at the fifth time level, $i=5$, and there are 10 possible $k$ and $a$ pairs). Each $k$ and $a$ combination applied to the average depth of infiltration $\bar{z}_i=V_Z(t_i)/x_A(t_i)$ led to a corresponding opportunity time $\tau_{oi}$. These were then numerically averaged. A straight line through a plot of $\bar{z}_i$ versus $\tau_{oi}$ on logarithmic paper provided a global representative $k$ and $a$ for the border. This technique allows viewing the infiltration function as it develops, not unlike data from rings.

Typical results are shown in Fig. 4 [from Clemmens (1982)] along with the best-fit Kostiakov function for the entire irrigation. These are contrasted with the results of the Fangmeier (Roth et al. 1974; Fangmeier and Ramsey 1978) and Gilley (1968) methods, as will be detailed later.

In a computational simplification, Clemmens guesses $a$, say 0.5, allowing a direct, rather than simultaneous, solution for $k$ at each $t$ up to $t_i$, with the same averaging of opportunity times leading to a point on the plot. The slope of the line, on logarithmic paper, provides a better guess for $a$, and the process is repeated until $a$ converges (Clemmens reports two trials typically necessary).

With the infiltration parameters found, Eq. (26) or Eq. (27) provides the changes in subsurface storage in each segment in each time step, while the depth hydrographs at the stations yield the changes in surface storage. A volume balance between stations and time steps yields the discharges at each station, while the profiles derived from the water-surface elevations yield the water-surface slope. Thus Manning $n$ can be determined at each station at each time level. Averaging, as in Clemmens (1982), yields a representative value for the border strip.

If the advance points leading to Eq. (26) had been found at constant increments of time (either measured or interpolated) as in Fig. 5, Eq. (26), with the Kostiakov power law, $z=k\tau^a$, leads to a numerical evaluation of $r_Z$, the summation below

$$r_{Zi} = \frac{V_{Zi}}{kt_i^a x_{Ai}} = \sum_{j=1}^{N}\frac{\Delta x_j}{x_{Ai}}\left(\frac{\{i-j+1\}^{a+1} - \{i-j\}^{a+1}}{i^a(a+1)}\right) \qquad (28)$$

in which $r_Z$=ratio of average depth of infiltration to the upstream depth. The relationships, $t_{Aj}=j\Delta t$, $t_i=i\Delta t$, $t_{Aj}-t_{Aj-1}=\Delta t$, etc., were used in the development of Eq. (28).

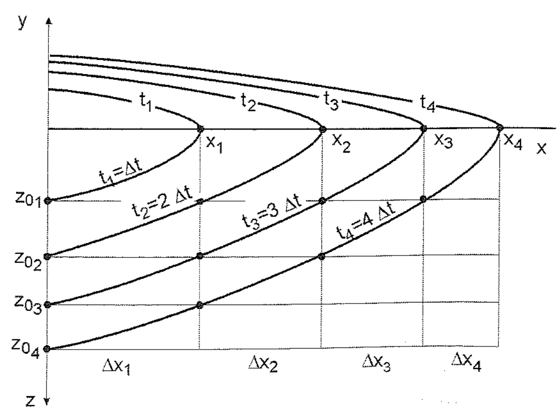



**Fig. 5.** Surface and infiltrated profiles at a sequence of times steps a constant interval $\Delta t$ apart

### *Measured Surface-Depth Hydrographs Replaced by Theoretical Values*

Monserrat (1994) devised a volume balance for the advance phase that used theoretical rather than measured values of average surface-water depths. He minimized an objective function, based on a balance between inflow, surface, and infiltrated volumes in level or sloping borders calculated with measured data, assumed Manning $n$, and unknown $k$ and $a$. The optimization procedure allows solution for both $k$ and $a$, as opposed to the inversion technique of Clemmens (1991), as well as $\overline{A_y}$; on the other hand, Monserrat's static database of solutions limits the analyses to the conditions, broad as they are, of the prerun simulations.

Monserrat (1994) calculated, theoretically, distance-averaged surface-water depths $\bar{y}$ in border strips, by interpolating within a specially prepared database, precalculated once and for all by numerical solution of the zero-inertia hydrodynamic equations. The Kostiakov power infiltration formula was used, restricting the inverse problem to this form as well. Interpolation within the database proved far more accurate than assumptions of a shape factor and normal depth upstream (Monserrat and Barragan 1998). The solutions and database are in dimensionless form to take advantage of the great data compression this allows. In essence, the database consists of tables of

$$\overline{y^*} = F(k^*, a, x_A^*) \tag{29}$$

in which the starred variables are dimensionless ratios of the pertinent dimensioned variable and a reference value

$$\overline{y^*} = \frac{\bar{y}}{Y_R}; \quad k^* = \frac{k}{Y_R/T_R^a}; \quad x_A^* = \frac{x_A}{X_R}; \quad t^* = \frac{t}{T_R} \tag{30}$$

The definition of the reference variables depends on the bottom slope of the border strip. For borders set on a nonzero slope, $S_0$, and with inflow rate $q_0$ per unit width

$$Y_R = \left(\frac{q_0 n}{\sqrt{S_0}}\right)^{0.6}; \quad X_R = \frac{Y_R}{S_0}; \quad T_R = \frac{Y_R X_R}{q_0} \tag{31}$$

while for irrigation of horizontal border strips, with cutoff time $t_{co}$, the easily solved implicit corresponding expressions are

$$Y_R = \left(\frac{q_0 n}{\sqrt{Y_R/X_R}}\right)^{0.6}; \quad Y_R X_R = q_0 T_R; \quad T_R = t_{co} \tag{32}$$

The database files consist of some 5,000 values of dimensionless average depth $\overline{y^*}(k^*, a, x^*)$. The 1998 paper presents the general appearance of the function for selected $a$ and $k^*$ for the level and sloping bottom cases.

In application to the estimation of field infiltration parameters (Monserrat 1994), an objective function for minimization was derived from the simple dimensionless volume balance, applicable during precutoff advance

$$S = \sum_{i=1}^{N} \left( t_i^* - \overline{y^*}(k^*, a, x_{Ai}^*) x_{Ai}^* - \int_0^{x_{Ai}^*} k^*(t_i^* - t_x^*)^a dx^* \right)^2 \tag{33}$$

in which $t_x = t_A(x)$. The time and advance data are measured values, put in nondimensional form with Eq. (30), the average surface-water depth is found from the database by interpolation, and the infiltrated volume is found by numerical integration. One additional term to be squared and added to the others in the objective function in Eq. (33) is given by the postirrigation volume balance, when all surface water has run off (totaling the volume $V_{RO_E}$), namely

$$t_{co}^* - \int_0^{L^*} k^*(t_{Rx}^* - t_x^*)^a dx^* - V_{RO_E}^* \tag{34}$$

in which $t_{Rx}$=measured recession time for point $x$. In a strategy reminiscent of the work by Ley in 1978 (T.W. Ley, "Sensitivity of furrow irrigation performance to field and operation variables," unpublished MS thesis, Department of Agricultural and Chemical Engineering, Colorado State University, Fort Collins, Colorado, 1978) [see text surrounding Eq. (44)], the time period in the irrigation between the time of cutoff (or the end of advance—whichever is smaller) and the time at which all surface water has drained off is not represented in the objective function; there is surface water on the field during this time, but the database does not contain the average depths of flow for that period.

The reference variables of Eqs. (31) and (32) are readily calculated with an *a priori* estimate of Manning $n$, and known inflow and bottom slope (or cutoff time). The corresponding $\overline{y^*}$ values are interpolated among the tabulated values with three-dimensional polynomial interpolation.

Monserrat minimized the objective function using a version of the Gauss-Newton algorithm specially adapted to functions, which can be expressed as sums of squares (Bartholomew-Biggs 1977). A line search at each step [see Fletcher (1980) and Gill et al. (1981), and also Press et al. (1992)], requires derivatives with respect to $k$ and $a$. Monserrat notes that calculation times are low, that the greatest accuracy is achieved when the data on recession time is included, and that no problems regarding uniqueness have been experienced.

### *Power-Law Advance*

Significant simplification in evaluating the Lewis and Milne integral in Eq. (21) follows the assumption of a functional form for advance as well as for infiltration. In particular, if infiltration and advance are both assumed to follow monomial power laws

$$A_Z = K\tau^a = K(t - t_x)^a$$

$$x_A = ft^h$$

$$\frac{dx_A}{dt} = hft^{h-1} \tag{35}$$

Now as pointed out by Hart et al. (1968), assumption of functional forms for both infiltration and advance, if coupled to the common assumption during advance that the average surface-water depth is constant, overconditions the problem mathematically and leads to a violation of mass conservation physically. On the other hand it is evident, from logarithmic plots of advance calculated with mass conservation intact [solution of the Lewis and Milne equation for advance, see Hart et al. (1968)], that over a relatively short time interval, perhaps less than a log cycle, departures of the advance function from a power law (straight line on logarithmic paper) are minimal. Conversely, overconditioning the problem as stated, over, say, a twofold distance (for example, from 1/2 of field length to field length), violates continuity by just a little, often justifying this approximate approach. Physically, this means that the average surface-water depth is in fact not constant, but varies a little, just enough to satisfy the mass balance.

Substitution of Eq. (35) into the integral in Eq. (21) yields a relatively simple result. Indeed, following another change of variable, $\alpha = t_x/t$, the result, during advance, is

$$V_Z = x_A A_{Z0} r_Z = x_A \cdot Kt^a \cdot h \int_0^1 (1-\alpha)^a \alpha^{h-1} d\alpha \qquad (36)$$

The structure of the integral shows that $r_Z$ is a function of $a$ and $h$ alone. Its expression, known as the beta function, is formally derived in Appendix I, which also encompasses both the extended Kostiakov formulation of Eq. (1) and the time periods before and after the completion of advance. Suffice to say at this point that $r_Z$ of Eq. (36) has a very a simple approximation, differing from the theoretical result by less than 1% in the range of $a$ and $h$ normally encountered, viz. (Christiansen et al. 1966)

$$r_Z = \frac{1 + a + h(1-a)}{1 + a + h(1+a)} \qquad (37)$$

Thus, the ratio of average depth of infiltration to upstream infiltration depth remains constant during advance. For furrows, the infiltrated volume per unit length $A_z$ is usually assumed related to infiltrated volume per unit area $z$ by a multiplier, a constant nominal wetted perimeter $W$.

DeTar (1989), also assuming power-law infiltration and advance, developed a simple approximation to the beta function, with the resulting $r_Z$ differing by no more than 2% from the Christiansen approximation. His approach is based on the assumption that, given Eq. (1) with $b$ and $c$ zero, it follows that $\bar{z} = k\bar{\tau}^a$, in which the barred quantities represent *distance* averages over the length of the advancing stream at any time. In view of the ease of calculating the theoretically correct $r_Z$, there seems no advantage to this approach.

On the other hand, as could be expected, the summation in Eq. (28) during a power-law advance would approximate the result, [Eq. (66), Appendix I], expressed as the beta function or its Christiansen counterpart, Eq. (37). Moreover, the shape factor $r_Z$ would be constant as long as the stream continued to advance. And the results for $k$ and $a$ would then be comparable to those of the Gilley technique, described in a subsequent section. After advance was over, the summation in Eq. (28) would approximate the shape factor characterized by the incomplete beta function (Abramowitz and Stegun 1964, Sections 6.6, 26.5).

### Infiltration-Parameter Estimation with Power-Law Advance

These techniques differ primarily in their treatment of surface volume—ranging from total neglect to careful measurements and analysis of depth hydrographs during the course of the irrigation. An early example of advance measurements leading to infiltration parameters is found in Christiansen et al. (1966). They fitted a straight line with slope $h$ and intercept $f$ to the measured advance curve on logarithmic paper. A similar plot of $\Phi_Z(t)$

$$\Phi_Z(t) = V_Z(t) - \frac{Bt}{h+1} = V_Q(t) - \overline{A}_y x_A(t) - \frac{Bt}{h+1} = gt^p \qquad (38)$$

in which the unknown $\overline{A}_y$ and $B$ were selected to make the fitted curve as straight as possible, with slope $p$, yields $a$ [Eq. (67), Appendix I], and an intercept $g$ from which $K$ is readily found

$$K = \frac{g}{fr_Z(a,h)} \qquad (39)$$

As noted earlier, $V_Z(t)$ is obtained from the field data by subtracting surface volume from inflow volume. As departures of measured $V_Z(t)$ from a power law can sometimes be attributed to an incorrect value of $\overline{A_Y}$; Christiansen et al. (1966) in the absence of any data on surface-flow conditions, suggested selecting such a value as will yield as closely as possible a power-law relation for total infiltrated volume. They also note that a concave-upward logarithmic plot of $V_Z(t)$ suggests a significant additional term, $B\tau$, in the infiltration formula. A value of final infiltration rate can be selected *a priori* and its contribution to the infiltrated volume subtracted to cause the remainder of the total-volume curve to be as straight as possible (see original paper). Christiansen et al. (1966) did not consider the constant term, $C$.

Smerdon et al. (1988) explored the consequences of neglecting surface storage. They also investigated the Soil Conservation Service (SCS) empirical advance formula, $t_A = b_1 x_A e^{b_2 x_A}$, in which $b_1$ and $b_2$ are constants. In one variant, a single measurement of depth at the upstream end and an assumed value of surface-stream shape factor $r_Y$ yield an estimate of average stream cross-sectional area. Alternately, normal depth is assumed at the upstream end and calculated with an assumed value of Manning $n$.

At the other extreme, Gilley (1968) coupled detailed measurements of $\overline{A_Y}(t)$ with the assumption of power-law variation for both (measured) advance and (unknown) infiltration [similar to the Christiansen et al. (1966) approach]. Thus, the infiltrated volume follows a power law as well. With Kostiakov infiltration, fitting the growth of infiltrated volume with a power law is tantamount to assuming the average surface-water depth varies just as necessary to satisfy continuity. The solution of Eqs. (67) and (39) leads to both $k$ and $a$. In practice, the Gilley method often leads to values of $a$ that are too small, with consequent underestimation of infiltration at the larger times (see Fig. 4 for example).

Fangmeier (Roth et al. 1974) extended Gilley's technique to subsequent phases of an irrigation, measuring surface-water depths throughout the course of irrigations in test borders (a gradual 20% increase in upstream depth over the first hour was not uncommon). The resulting parameter-estimation technique was designed to utilize the volume balance after advance as well, up to inflow cutoff, in order to arrive at a single pair of Kostiakov $k$ and $a$ for the entire irrigation. The irrigation is subdivided into a number of time periods, $i$. At the end of each period, best estimates of $k$ and $a$ are updated, with the final values, after inflow ends, considered representative of soil conditions. During advance, the Gilley (1968) technique yields a shape factor $r_Z$ [Eqs. (66) and (37)], which together with measured average depth of infiltration provides $z(t_i)$ at the head of the border. At each $i$, a best-fit power function yields updated values of $k$ and $a$. Not surprisingly, these agree pretty well with the determinations resulting from the Gilley method. After advance is completed, at each successive time level an infiltration-depth profile is computed from the measured opportunity times and last best estimates of $k$ and $a$. Numerical integration yields an updated $r_Z$, which when applied to the measured average depth, provides successive $z(t_i)$ at the head of the border for participation in the power-law fit. Fig. 4 contrasts results of this method with Clemmens' linear station advance and Gilley's method.

For furrow flow, Fangmeier and Ramsey (1978) evaluated soil intake (volume per unit length) as the product of infiltration depth (volume per unit area) and a characteristic width—wetted perimeter or top width. The writers' method for determining $k$ and $a$, representative for both the advance and continuing phases, was based on an observed correlation, an approximate proportionality, between $a$ and the slope of the logarithmic plot of total infiltrated volume (calculated from the measured opportunity times and current estimates of $k$ and $a$) as a function of time. An examination of Fig. 9 (Appendix I), based on power-law advance, shows that during the advance phase, the infiltration and advance exponents,

$a$ and $p$, are in the constant ratio $a/(a+h)$, since $r_Z$ is constant, but that well into the continuing phase, $p$ approaches $a$, as $r_z \rightarrow 1.0$ [see Eqs. (35)–(37), (60), and (61)]. Fangmeier and Ramsey applied the observed ratio to the $p$ value exhibited by measured infiltrated volume (inflow minus outflow minus change in surface storage) to estimate $a$, while $k$ was chosen to make computed and measured total infiltrated volumes agree when recession starts.

### *Assumed Average Surface-Water Volume per Unit Length (Cross Section Area)*

Because of the intense data collection required for measuring the volume on the surface as the irrigation progresses, suggestions have been made for estimating it instead. A common assumption is that the bottom slope is steep enough that the upstream depth rises quickly to normal depth and remains at normal throughout the analysis. Thus the upstream cross-sectional area $A_{Y0}$ is estimated. A typical concomitant assumption specifies a known constant shape factor $r_Y$, usually guessed in the vicinity of 0.75, relating average surface-flow cross-sectional area to upstream cross-sectional area

$$\overline{A_Y} = r_Y \cdot A_{Y0} \tag{40}$$

This simplifying assumption artificially decouples the surface-flow characteristics from the infiltration upon which they, strictly speaking, depend.

With Eq. (40) and an assumed $r_y$ providing the surface storage during advance, and with the simple infiltration formula $Z=C+B\tau$ suitable for cracking soils, Mailhol and Gonzalez (1993) related stream advance to the infiltration parameters through a Laplace transformation of the Lewis and Milne equation (particularly simple for the given infiltration formula).

Monserrat and Barragan (1998) calculated dimensionless upstream depths and surface shape factors for a wide range of conditions in border and basin irrigation, providing a theoretical estimate of the errors introduced. Depth measurements in the field (Esfandiari and Maheshwari 1997b) showed instances of shape factors well in excess of 1.0. Some theoretical confirmation of surface profiles during advance exhibiting larger depths in the middle of the stream than at the upstream end (stemming from reductions in Manning $n$ with time, as the soil surface is smoothed by the flowing water) is provided in Clemmens et al. (2001). Alternately, $\overline{A_y}$ is calculated by solving the hydrodynamic equations of motion of the surface stream in a process that simultaneously determines one infiltration parameter to which the hydrodynamic equations are coupled [Clemmens (1991), discussed in greater detail later].

Actually there are two aspects to the inverse analysis of the advance phase (parameter estimation as opposed to calculation of advance) inherently coupled to each other, viz., determination of $\overline{A_y}$ and extraction of the infiltration parameters such as $k$ and $a$ in the course of the analysis. The behavior of the surface stream affects the infiltration, while the infiltration plays a strong role in determining the characteristics of the surface stream. Some of the various methods proposed, including those that artificially decouple these two aspects (such as the aforementioned assumed normal depth and shape factor), are described in the remainder of this section.

#### Two-Point Method (Elliott and Walker 1982)

In a popular variation of the Christiansen et al. (1966) approach, advance time is measured only twice, typically half-way to the end of the field and at field end. The method assumes power-law advance and, rather than base $\overline{A_Y}$ on the degree to which $V_Z$ follows a power law, makes an independent estimate for $\overline{A_Y}$, based on assumed normal depth for the given cross section, bottom slope, Manning $n$, and inflow rate [Eq. (4)] and an assumed constant shape factor $r_Y$. Then $V_Y$ in Eq. (5) is known at both $x_A$. The two-point method shares with similar methods any inaccuracies stemming from the estimation of the average cross-sectional area of the surface stream (Strelkoff et al. 2003). It also shares the small violation of mass conservation stemming from assumption of power laws for both infiltration and advance along with a constant average cross-sectional flow area during advance. Concern that one or both of the two advance measurements may be faulty can be alleviated by plotting a line of best fit to the advance curve on logarithmic paper and determining the two points from that. The two-point method is easily extended to accommodate nonzero $b$ values in Eq. (1), once these are determined independently (see Appendix I). Note, however, the caveat following Eq. (3). The outcome of the method are the Kostiakov $k$ and $a$, as follows:

$$h = \frac{\log 2}{\log\left(\dfrac{t_L}{t_{L/2}}\right)}; \quad f = \frac{L}{t_L^h} \tag{41}$$

$$p = a + h = \frac{\log\left(\dfrac{V_{ZL}}{V_{ZL/2}}\right)}{\log\left(\dfrac{t_L}{t_{L/2}}\right)}; \quad g = \frac{V_{ZL}}{t_L^{a+h}} \tag{42}$$

with Eq. (39) yielding $K$.

Fig. 6 illustrates the effect of an incorrect estimate of average surface-stream cross section for two different bottom slopes: (1) $S_0=0.0005$; (2) $S_0=0.005$. The two-point method was applied to a simulated border irrigation. The infiltration-function input to the simulation (a time-rated family member requiring 4 h to infiltrate 100 mm) is labeled SRFR; the simulation then yielded both an advance curve ($t_2$ shown) and $r_Y$, slowly varying with time. A representative value was selected for use in the two-point method, and the resulting infiltration function is labeled "$r_Y$ correct." Then the representative value was changed by $\pm 10\%$ with the results shown. As can be expected, with the surface volume at the small slope comprising a greater fraction of the total inflow, an error in $r_Y$ leads to greater errors in estimated infiltration than with the large slope. In both cases the deviations from the correct infiltration increase significantly for times extrapolated beyond $t_2$.

In the event of a level field, normal depth has no meaning, but an extension of the method to this case has been proposed (Zerihun et al. 2004). Instead of assuming $A_{Y0}$ constant in Eq. (40), it is allowed to grow with $x_A$ in accord with Eq. (4) and the approximation that $S_f=y_0/x_A$. Then $A_{Y0}$ is implicit in the relationship

$$A_{Y0}^2 R_0^{4/3} y_0 = x_A Q_0^2 n^2 \tag{43}$$

derived from Eq. (4) and the furrow cross-sectional shape; the right-hand side is known at both advance distances [selected in Zerihun et al. (2004) as L/3 and 2L/3]. Then $\overline{A_Y}$ follows from Eq. (40). To extend the method to nonzero $B$ requires depth measurements as the ponded water surface falls after cutoff. $B$ would be given by the measured rate of fall and the assumption that at this point in time, infiltration rate does not vary with location in the basin.

Alvarez (2003) used the assumptions of the two-point method to predict the advance and Kostiakov $K$ for furrow discharges other than the particular one at which the field test was run, to

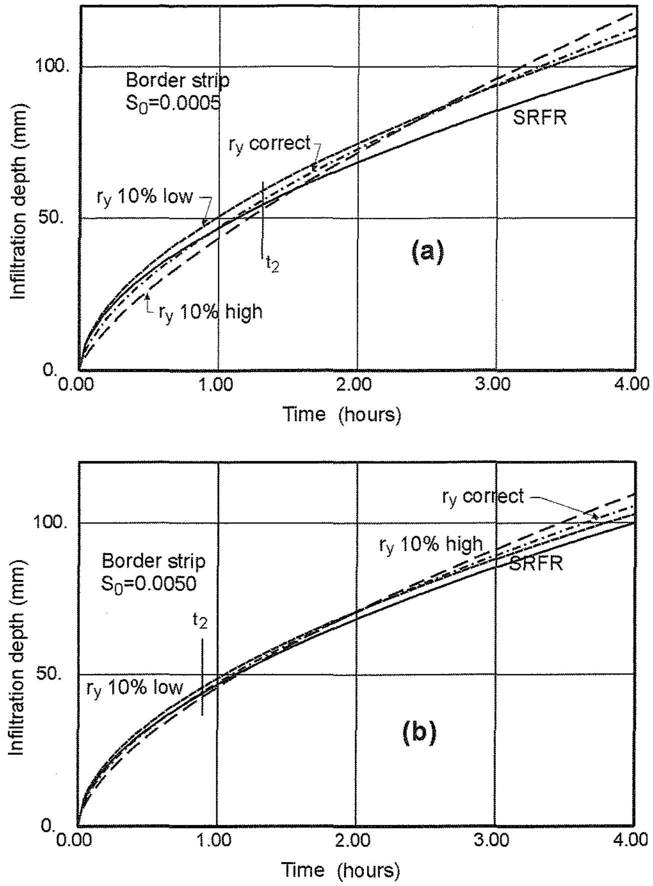


**Fig. 6.** Effect on predicted infiltration function due to variation in estimate of average surface-stream cross section. (a) low bottom slope: $S_0$=0.000 5; (b) high bottom slope: $S_0$=0.005. $t_2$=time to advance to field end. Two-point method (Strelkoff et al. 2003).

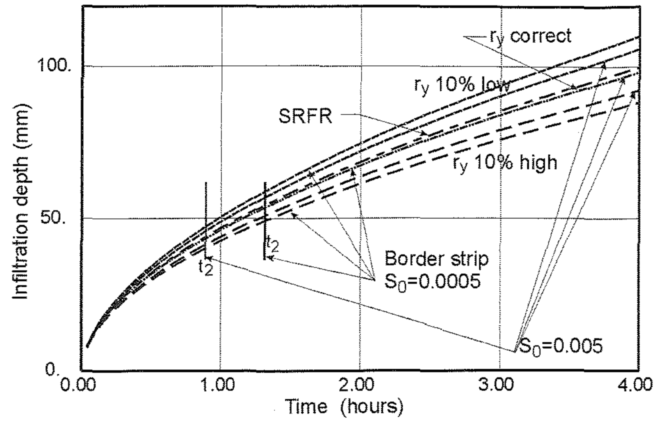


**Fig. 7.** Effect on predicted infiltration function due to variation in estimate of average surface-stream cross section. (a) low bottom slope: $S_0$=0.000 5; (b) high bottom slope: $S_0$=0.005. $t_2$=time to advance to field end. One-point method (Strelkoff et al. 2003).

account for the different wetted perimeter. The technique is based on the further assumption, supported by field observations, that with different inflow rates, the advance exponent varies very little.

## One-Point Methods

One-point methods require measurement of time to advance to a single point, typically at field end. It is assumed also in this method, as usually applied, that the flow channel is sufficiently steep to achieve normal depth at the upstream end quickly, and that $r_Y$ remains essentially constant during advance. This allows solution of the Lewis and Milne (1938) integral equation. And, while two measured advance points are generally required to yield independent values of the two Kostiakov parameters, additional assumptions can yield the pertinent parameters with just one advance point. In the original one-point method (Shepard et al. 1993), the reasonable assumption of a Philip infiltration function, $z=S\tau^{1/2}+A\tau$ (Philip 1969), and a quite restrictive advance function, $x=\mathrm{f}t^{1/2}$, lead to both constants, $S$ and $A$, from a measurement of the time to reach furrow end. The concurrent assumptions, instead, of NRCS (SCS) infiltration families and power-law advance, $x=\mathrm{f}t^{h}$, with $f$ and $h$ site-specific constants, lead approximately to the pertinent family, once the time for the stream to reach a specified point is given (Valiantzas et al. 2001). This is made possible by the fact that each family is defined by a particular value of $k$ and a particular value of $a$; thus $k$ is a function of $a$. In an alternate method, introduced because some noncracking soils are fit better by the time-rated families of Merriam and Clemmens (1985), a similar development (see Appendix II) also leads to Kostiakov $k$ and $a$ from a single advance point. As noted in Appendix II, the empirical relationships between $k$ and $a$ depend on the actual wetted area, and consequently, in furrows $k$ should be based on wetted perimeter rather than on furrow spacing.

A comparison between behaviors of the one-point (with time-rated families) and two-point methods under a limited range of conditions is found in Strelkoff et al. (2003) and is illustrated by comparing Figs. 6 and 7. Fig. 7 shows the effect of varying $r_Y$ in the one-point method for the same soil and slopes as in Fig. 6. In both, errors decrease with increasing slope as surface volumes decrease in relative size. One-point errors increase gradually and predictably as time is extrapolated beyond $t_2$. As $r_Y$ is changed, $k$ and $a$ change in the same direction. Trends of the two-point errors are difficult to predict. Changes in $r_Y$ lead to $k$ and $a$ changes in opposite directions. Error magnitudes are comparable. If extrapolation beyond $t_2$ is reduced (with advance occupying a greater fraction of the total run time) the errors in the two-point method are reduced.

In general, to the extent that the Merriam and Clemmens (1985) observation is correct, viz., that infiltration in noncracking soils can be characterized by the time it takes to infiltrate 100 mm, the one-point method described does an excellent job of estimating the entire infiltration versus time function, given the time for a stream to advance to field end. Error in the estimate of average cross-sectional area of stream flow affects the results in a moderate and predictable way. Worthy of note, however, the time-rated families do not extend to Kostiakov $k$ values below about 35 $\mathrm{mm/h}^{a}$.

For soils that do not fit the time-rated families very well, for example with a given $a$, and $k$ smaller than the corresponding value for the family (not shown), the errors in predicted infiltration increase substantially, with both correct and incorrect $r_Y$. Extrapolation errors are very significant. Increased bottom slope reduces the error. The one-point method fails altogether for a soil as heavy as assumed and for excessive assumed surface-water volumes. With $k$ larger than that given by the families and little extrapolation, the errors are relatively minor.

The two-point method fares better than the one-point method when $k$ is smaller than the family value, though extrapolation errors are still significant. It does not fail for small $k$, but errors are substantial with excessive assumed surface volume. Increased bottom slope reduces the error. With $k$ larger than that given by the families and little extrapolation, the errors are relatively minor.

With a given inflow volume at the time, $t_2$, any error in assumed surface volume is directly applied to the resultant assumed infiltrated volume. For example, if at $t_2$ the surface and infiltrated volumes are about equal (e.g., in Fig. 7 with $S_0=0.0005$), a 10% assumed decrement in surface volume leads to about a 10% increase in estimated infiltrated volume, and hence in predicted $z(t)$. If, however, because of larger bottom slope or smaller Manning $n$ the surface volume is only one-half of the infiltrated volume ($S_0=0.005$), a 10% error in surface volume leads to only a 5% error in $z(t)$. These considerations evidently extrapolate to times well in excess of $t_2$.

With the two-point method, the effect of relative errors in $\overline{A_y}$ on $z(t)$ also depends on the relative sizes of surface and infiltrated volumes at the times of measurement, but the regular behavior appears limited to times less than $t_2$. With the degree of freedom characteristic of the method, extrapolation beyond $t_2$ can increase the error in $z(t)$ considerably, and also reverse the direction of the effect of errors in $\overline{A_y}$.

These conclusions must be considered tentative in view of the limited range of conditions investigated. Field length, for example, was limited to 200 m. Furthermore, no basic infiltration rate was considered for the soils, and infiltration was assumed to follow the Kostiakov power law exclusively.

At best, results based on advance data alone apply principally to the time period of advance. There can be significant changes in infiltration rates at greater times, not well predicted by behavior during advance. In particular, most soils exhibit a basic or final, essentially constant infiltration rate at large times, often greater than the advance time. To capture the influence of longer time periods upon infiltration-parameter estimation, a number of techniques have been proposed using not only a measured advance curve, but a measured runoff hydrograph as well.

Scaloppi et al. (1995), focusing on sloping-furrow irrigation, sought a relation in which the Kostiakov $k$ and $a$ could be determined by simple regression. Therefore, they found it necessary to assume a functional form for the advance, again, a power law. And, while recognizing the possibility of measuring surface-stream volumes at a sequence of advance times (through knowledge of the dry-furrow cross-sectional shape and measured wetted top width at evenly spaced stations), they proposed approximating assumptions leading to formulas for estimating surface-stream volume during the advance and runoff phases in terms of an assumed Manning $n$, and an assumed shape for the surface-water profile. This is based on the assumptions of (1) a power-law relationship between top width and depth in the cross section, (2) normal depth at the upstream end, and, during advance, (3) a power-law relationship between depth and distance back from the leading edge of the stream, with an assumed power, typically, 0.35. Once runoff has begun, normal depths based on periodically measured rates are assumed at the downstream end of the furrow. The stream profile is then assumed to consist of two branches: the aforementioned power law in the upstream portion, and uniform flow at the runoff normal depth in the downstream portion, the distance at which the power-law depth matches the downstream normal depth constituting the crossover point; the volume under the profile is calculated accordingly.

The infiltration parameters sought are $k$, $a$, and $b$, in the assumption that volume infiltrated per unit length is proportional to a characteristic width $W$, the space-averaged wetted perimeter, or as suggested, top width of the flow. Advance is assumed to follow a power law in time so the subsurface shape factors are given by Eq. (37) during advance, and by Eq. (68) (Appendix I) after. The basic infiltration rate $b$ is estimated from inflow/outflow measurements, while regression analysis ultimately yields $k$ and $a$.

Ley (1987) avoided postadvance assumptions on the surface-water profile by hopping over this period, connecting again with field behavior after recession. In a search for $a$, $k$, and $b$, Ley (1978) modified the approach of Christiansen et al. (1966) to include matching predicted and observed total runoff volume as well as the advance function. A Fibonacci search procedure for $b$ in Eq. (1) (Kahaner et al. 1989) was used to minimize the differences. At any value of $b$, with measured advance assumed to follow a power law, $x_A=\mathrm{f}t^h$, and with $\bar{y}$ estimated from $q_0$, $S_0$, and assumed $n$ and shape factor $r_Y$, the mass balance (e.g., for a border) can be written

$$r_Z(a,h)kt^a=\frac{q_0t}{x_A}-\frac{bt}{1+h}-\bar{y} \tag{44}$$

Slope and intercept of a logarithmic plot of the right-hand side with time provides $k$ and $a$ for that choice of $b$. With the measured advance and recession curves, these infiltration parameters yield a computed infiltrated volume. The search for $b$ continues until the difference between this computed volume and the difference between measured total inflow and outflow volumes is minimal. Of interest, Ley recommended (without comment) using only the last one-half of the advance curve to fit with the power law. In furrows, recession was often assumed instantaneous, at the time of cutoff.

More precise estimates of surface volumes can be achieved by simulating the irrigation, i.e., solving the governing partial differential equations of mass and momentum conservation for the given conditions. The hydrodynamic equations governing surface-irrigation flow are not based on any assumptions on the size or shape of the surface stream. Instead, the stream is subdivided into segments and equations of mass and momentum conservation are written for each [see, e.g., Walker and Skogerboe (1987)]. While Eq. (5) is presented for the entire length of the irrigation stream, in principle it is applicable to any segment, with $Q_0$ referring to the inflow to the segment and $Q_{RO}$ to the outflow, etc. Intake can be calculated on the basis of opportunity time and wetted perimeter, given an empirical infiltration formula. A second equation is based on the equilibrium of forces (velocities in surface irrigation are so low that accelerations are wholly negligible) comprising the downslope component of weight, the hydraulic drag of soil surface and plant parts, and unbalance in hydrostatic pressures on the front and rear faces of a segment. When these equations are complemented by a known inflow and the fact that flow depths and discharges are zero at the advancing front, there are just enough equations to numerically solve simultaneously for surface-water depths and discharges everywhere, as well as advance, infiltration, runoff, and recession in a complete simulation [WinSRFR, e.g., E. Bautista et al. (2009); SIRMOD, Walker (2004)]. The results are as applicable to a level, or irregular basin as to a steeply sloping furrow.

### Direct Inversion of the Hydrodynamic Flow Equations

Clemmens (1991) was able to invert the hydrodynamic unsteady-flow equations governing the irrigation-stream flow to the extent of one unknown field parameter. In the double-sweep algorithm for solving the linearized equations of the Newton-Raphson scheme [see, for example, Walker and Skogerboe (1987)] if the time step is given and the advance increment is the unknown, then the sparse matrix of coefficients that is inverted at each iteration in each time step to yield the solution for the depths and discharges in the body of the stream will have nonzero values clustered only about the diagonal. But in the event the time step is unknown and the advance increment is given, the matrix exhibits an additional column of values. An extension of the traditional double-sweep technique for the matrix inversion, accounting for the additional column, was provided in Strelkoff (1990,1992). When, then, a column of coefficients stemming from the gradients of depth and discharge with respect to some field parameter (Clemmens 1991) is placed in the matrix in place of the one expressing the gradients with respect to an unknown time step, and both time step and advance increment are specified (from measurements), that field parameter comes out in the solution at each time step.

The method requires *a priori* specification of all the remaining field parameters, usually, Kostiakov $a$ and Manning $n$. At each time step, the resultant $k$ is used to calculate an opportunity time for the computed average depth of infiltration. Then the resulting plot is used to fit a new Kostiakov function—allowing recalculation with the more current value of $a$. The original paper provides a wealth of detailed comparisons of results from various combinations of assumptions in the method.

## Parameter Optimization through Repeated Simulation

When the unknown field parameters are so embedded in the pertinent equations that the latter cannot be inverted to yield them, they can be sought in a program of successive approximation. This typically involves repeated simulations, with changing values of the parameters, in a formal search procedure known as optimization, aimed at minimizing the discrepancies between simulated and measured values of selected quantities, such as advance, water depths, runoff hydrographs, etc.

General concerns and approaches to secure an optimum can be found in Press et al. (1992) Chapter 10, which explains the concepts of much of what follows here—in detail and with much background information. In particular, they first discuss the problems encountered and techniques available in a one-dimensional search for a minimum. The global, or true, minimum of a general objective function $F(x)$ of one independent variable $x$ can certainly be found, within a prespecified permissible range of the independent variable, by brute force, i.e., calculating $F$ over sufficiently small increments, $\delta x$, that a dip lower than all the others will not be missed. The fundamental problem in practical one-dimensional optimization is in devising a technique that will lead to the global minimum via a reasonable number of calculations. In multidimensional optimization, in which there are a number of independent parameters, described, say by the vector $\mathbf{p}$, the additional problem to be confronted is in the selection of directions in which to change $\mathbf{p}$ from a current estimate, as well as in how much to change it, so as to reach a global minimum, all within the permissible range of $\mathbf{p}$. In general, a good optimization algorithm is robust, i.e., will inexorably converge to the global minimum, bypassing local minima, regardless of the starting guesses for the independent parameters or the nature of the objective function. It will be economical of computation and will not require excessive computer storage.

In the *gradient methods*, the search proceeds in a series of one-dimensional steps, with the direction in each step selected in response to the gradient of $F$ (calculated, for example, by simulation at two neighboring $\mathbf{p}$).

In the step from iteration $i$ to $i+1$

$$\mathbf{p}^{(i+1)} = \mathbf{p}^{(i)} - \mathbf{d} = \mathbf{p}^{i} - \alpha^{(i)}\mathbf{R}^{(i)}\mathbf{g}^{(i)} \tag{45}$$

the change in $\mathbf{p}$ is made in the direction of the vector $\mathbf{d}$, given by the matrix $\mathbf{R}$ premultiplying the gradient $\mathbf{g}$, and of length given by the scalar $\alpha$. $\mathbf{R}=\mathbf{I}$, the identity matrix, corresponds to the method of steepest descent, but this can be extremely inefficient (Press et al. 1992), and other choices, twisting the direction of correction away from the gradient, prove more suitable in the search for the bottom of a long narrow valley with spurious local minima representing the objective function (as visualized with two free $\mathbf{p}$ components). A logical selection for $\mathbf{R}$ in the search for the minimum of the $F$ surface is the inverse $\mathbf{H}$ of the Hessian matrix for $F$

$$\mathbf{H} = \left(\frac{\partial^2 F}{\partial p_i\, \partial p_j}\right)^{-1} \tag{46}$$

for that is an indication of the local rate at which the gradient is approaching zero, i.e., where $F$ is at a minimum. The expectation is that, except for points at the boundaries of the permissible domain of independent variables, the minimum of the objective function corresponds to a zero gradient there. Direct calculation of $\mathbf{H}$ is inconvenient, partly because of the difficulty in determining the second derivatives and partly because of the need for inverting a potentially large matrix. A number of numerical alternatives have been proposed, as noted in subsequent sections.

### *Optimization to Match Advance*

A series of methods, also, in a sense, constituting an inversion of the advance problem, consists of repeated simulations with a mathematical model controlled by one or another search procedure. Walker and Busman (1990), with the aim of evaluating real-time infiltration, i.e., while an irrigation was in progress, minimized an objective function, $Y$, based on measured and calculated advance

$$Y = \sum_{j=1}^{m} \sqrt{(T_{Aj} - t_{Aj})^2} \tag{47}$$

in which the $T_{Aj}$=measurements of advance time to specified locations $x_j$; while the $t_{Aj}$ are simulated times with specific values for the infiltration parameters, in a current estimation. The roughness parameter is assumed to be known. The number of comparison points $m$, increases as the irrigation progresses, providing ever more data on which to base the infiltration parameters. At any step of the irrigation $m$, the objective function is minimized (the infiltration parameters are optimized) in successive approximations via the downhill simplex method (Press et al. 1992), not requiring calculation of the gradient of the objective function, but only of the objective function itself, at $n+1$ estimates of the $n$ infiltration parameters at each step of the iterative process, i.e.

$$Y_i = \sum_{j=1}^{m} \sqrt{(T_{Aj} - t_{Ai_j})^2}, \quad i = 1, \ldots, n+1 \tag{48}$$

With two infiltration parameters $k$ and $a$ ($n$=2), there are three vertices to the simplex, in general a polyhedron with $n+1$ vertices in $n$-dimensional parameter space, reducing, in the case of Kostiakov parameters, to a triangle. With three, in a search for best values of $k$, $a$, and $b$, the simplex is a tetrahedron in three-dimensional space. Each vertex is characterized by a set of values for the desired $n$ parameters. The simplex method consists in a strategy (Press et al. 1992) for changing the locations of the vertices in a manner designed to bring the optimum point into the interior of a very small polyhedron.

Azevedo (1992) found the multidimensional modified Powell method for determining successive directions for solution-vector change coupled to the one-dimensional line minimization of Brent at each direction change, together with minimum bracketing [all in Press et al. (1988)] much faster and more reliable than the downhill simplex method. He also found that smaller minimums of the objective function prevailed when all three Kostiakov-Lewis parameters—$k$, $a$, and $b$—were sought in the search; field determinations of $b$, for example, were often in considerable error. Of note, the solutions proved not unique, with different ultimate parameter values resulting from different starting values. At the same time, advance calculated with the different sets did not differ much.

Bautista and Wallender (1993) used the Marquardt method (Press et al. 1992), a standard technique for nonlinear least-squares curve fitting, to minimize the objective function

$$F(\mathbf{p}) = \sum_{j=1}^{N} \left( \frac{T_{Aj} - t_A(\mathbf{p})_j}{\sigma_j} \right)^2 \tag{49}$$

in which $T_{Aj}$=measured advance to a given station $x_j$; while $t_{Aj}$ is simulated, with field-parameter vector $\mathbf{p}$ having components, $k$, $a$, and $b$; and $\sigma$=weighting factor, generally set to unity, to account for uncertainty in a measurement. Roughness was assumed known, and ultimately, $b$ also (field-measured steady-state infiltration rate), as the Marquardt algorithm exhibited difficulty in converging to optimum values of three parameters. An alternate objective function, couched in terms of advance velocity, proved more useful in identifying spatially varying infiltration, but ultimately, those results were inconclusive.

All of these methods provide more reliable infiltration data if advance data from the entire irrigation is used, rather than any portion of the advance curve, e.g., in an attempt to discern the infiltration parameters as the irrigation progresses.

### Optimization to Match Outflow Hydrographs

Gillies and Smith (2005) used a robust volume-balance scheme to determine infiltration parameters in a modified Kostiakov formulation. Advance was assumed to follow a power law as well. Both advance and runoff data were matched in calculating least squares of the differences. The procedure was subsequently amended to account for inflows different from those measured in the test furrow (Langat et al. 2007) [leading to differences in wetted perimeter, as in the modifications to the two-point method proposed by Alvarez (2003)] and for temporal variation in the measured inflow hydrograph (Gillies et al. 2007).

Walker (2005) developed a systematic procedure for selecting appropriate values of $K$, $a$, $B$, and Manning $n$ in a simulation that would match measured outflow hydrographs, when measured inflow, furrow cross sections, and bottom slope were given. The search procedure differs from classical optimization of several parameters simultaneously in that the parameters are sought sequentially, albeit iteratively. The concept of sequential determination of the parameters is based on the general observation of a wide range of simulations that the four sought-after parameters affect the results in different ways, specifically, that the $K$ value is the most influential in determining advance time (the time outflow begins), that $a$ is the most important factor affecting the shape of the outflow hydrograph, while $B$ is the most important in determining the absolute values of runoff rate. The roughness $n$ was seen as most influential in determining the time recession ends (the time outflow ends). The necessity of iteration stems from the imprecision of these observations.

In the multiple simulations comprising the method, the measured furrow geometry and inflow hydrograph are augmented by successive approximations to the four parameters. Starting guesses follow from Eq. (3) for $B$, the two point method [Eqs. (39), (41), and (42)] for $K$ and $a$, and values in the literature for $n$. In the sequence of nested optimizations, $K$ is adjusted in the innermost nest of simulations to make computed and measured advance times equal. The secant method provides the successive approximations as changes in $t_L$ are nearly proportional to changes in $K$. In the next outer nest, a Fibonacci search (Kahaner et al. 1989) for $B$ minimizes $\text{rms}_{\text{RO}}$ the RMS of the differences between computed and measured runoff rates

$$\text{rms}_{\text{RO}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (Q_{\text{RO}iM} - Q_{\text{RO}iS}|_{\hat{K},\hat{a},\hat{B},\hat{n}})^2} \tag{50}$$

where $N$=number of points in the hydrograph; the subscript $M$ refers to measured values; $S$ refers to simulated values; and the hat above a variable, e.g., $\hat{K}$, refers to the current value that satisfies the search. At each value of $B$ found in the process, the inner nest of searches adjusts $K$ to match the advance time. The next ring of the search is for $a$, which also minimizes the RMS of the computed and measured values of runoff. At each a value, $\hat{B}$ and $\hat{K}$ are recomputed. The final, outer ring determines $\hat{n}$ to match computed and measured recession times. A key advantage of the method is that all data collection during the irrigation is confined to the inlet and outlet of the furrow, with no need to enter upon the wet field.

The issue of how much information about the field parameters can be deduced from the measurements of advance alone (observability of advance) was examined theoretically, in the context of a linearized zero-inertia model by Katopodes (1990). He concluded that resistance and infiltration have indistinguishable effects on the rate of advance, but that if one is given, the other can be found. Of the three parameters, Kostiakov $k$ and $a$ and Manning $n$, if two are known, the third can uniquivocally be deduced from the observed advance. Two parameters can be deduced if measured surface depths are available for comparison. The optimization can be difficult; in the case of an objective function based on varying $n$ and $a$, several local minima are observed, in a very narrow valley, each of which is higher than the global minimum. All three parameters can be estimated from measurements of a single depth profile (Yost and Katopodes 1998).

### Matching Measured Depths

An estimation technique based on a few selected depth measurements at various locations or times during advance (not enough to define stream volume, but only to compare with computed depths) was presented by Katopodes et al. (1990). The objective function to be minimized at the optimum values of the field parameters, presented as a vector $\mathbf{p}$ with components $n$, $k$, and $a$

$$\mathbf{p} = \begin{bmatrix} n \\ k \\ a \end{bmatrix} \tag{51}$$

is

$$F(\mathbf{p}) = \sum_{j=1}^{J} [Y_j - y(x_j, t_j, \mathbf{p}\ ]^2 \tag{52}$$

in which $Y_j$=depth measurement at some $x_j$, $t_j$ and $y$ =corresponding computed point in a simulation with field parameters $\mathbf{p}$. In principle, the components of $\mathbf{p}$ can each have components reflecting spatial variability, this increasing the total number of unknowns; in general, the greater the number of unknowns, the more problematic the search. To avoid unacceptable (unrealistic) $\mathbf{p}$ values, the objective function is augmented with constraints, which cause the objective function to grow very large when these are close to being violated, and so drive the search away from those values.

In the gradient method selected by Katopodes et al. (1990), two modifications obviating the need for any direct calculation of the Hessian were proposed, both requiring calculation of first derivatives of the objective function—a conjugate gradient method and a variable metric method (Press et al. 1992). Both methods utilize line search, a one-dimensional search for a function minimum in a multidimensional parameter space, the search made one dimensional by specifying the direction of the vector $\mathbf{d}$ of parameter changes. In essence, the methods utilize information, collected in successive steps of descent, to approach an approximation to the Hessian, and so indicate the direction of the line search in the next step. They also have features intended for identifying and disregarding local minima in the quest for the global minimum.

Katopodes et al. (1990), by using a single depth profile with about 10 points in Eq. (52), were able to search out rapidly and without difficulty the appropriate values of $n$ and $k$, if $a$ were fixed, or $a$ and $k$ if $n$ were fixed. Searches for all three parameters would converge only if starting values were fairly close to correct. This problem was corrected in Yost and Katopodes (1998) through introduction of another search procedure, a so-called fixed point method, which converges slowly, but inexorably. They recommend starting the search with the new procedure and switching to a gradient method as the parameters hove into range.

Several similar gradient procedures were applied by Playán and García-Navarro (1997) in a search for the $n$-$k$ parameter pair and also the $a$-$k$ pair, with the third component of the parameter vector held fixed. They used code made available in Press et al. (1988). Of the various techniques, the best choice proved much the same as that used in Katopodes et al. (1990) and identified by Press et al. (1992) as the Polak-Ribiere method, but with centered finite-difference approximation to the gradient, rather than a forward difference. Exhibiting somewhat better convergence characteristics in the more problematic search for the $a$-$k$ pair was the Powell method [see Press et al. (1988,1992)]. Playán and García-Navarro (1997) applied these parameter-identification techniques to a total of about 60 depth measurements over some six time periods in the objective function. Both one-dimensional level basin flow and purely radial flow, independent of the polar angle, from a corner inlet were investigated; the results were essentially equally good.

Both Playán and García-Navarro (1997) and Katopodes et al. (1990) in their research used simulated stream lengths and depths as the source for "field" measurements, and both extend the caveat that use of truly measured values has not been tested. In particular, one could expect difficulties stemming from spatial variability, measurement error, and in general, data which is not smooth.

### Matching Measured Advance and a Depth Hydrograph

A quite different procedure for estimating whole-border infiltration and roughness for the advance phase was developed by Valiantzas (1994) using a combination of measurements and theoretical (zero-inertia) simulations (e.g., Strelkoff and Katopodes 1977). The field parameters sought are the Kostiakov $k$ and $a$ and the Manning $n$ (actually its equivalent, normal depth $y_N$ for the given inflow and bottom slope). Both the advance trajectory $x_A(t_i)$, $i=1,\ldots,N$, and a depth hydrograph $y_R(t_j)$, $j=1,\ldots,M$, in a reference point $x_R$ at or near the inlet end of the border, are measured. The procedure requires a series of iterative steps each of which employs simple volume balances equipped with theoretical shape factors and depth hydrographs derived from simulations based on current values of $k$, $a$, and $n$. The volume balances, in turn, yield $k$ and $a$ for the simulations, while linear regression between depth values and zero-inertia predictions yield the required $n$.

The volume-balance equations with the unknown (constant) infiltration parameters, $k$ and $a$, are written in the form

$$F_A(k,a) = \sum_{i=1}^{N} \left[ x_A(t_i) - \frac{q_0 t_i}{r_Y(t_i) y_R + r_Z(t_i) k t_i^a} \right]^2 \tag{53}$$

in which $r_Y$ and $r_Z$=current values based on theoretical simulation and the $t_i$=times at which the advance comparisons are made. Evidently, $r_Y$ is a little different from its standard meaning, being the ratio of average stream depth to the reference depth, $y_R$, not necessarily exactly at the upstream end of the border. Eq. (53) requires nonlinear least-squares fitting [the writer suggesting a linearization method (Draper and Smith 1981)], which may be worthwhile as the final values of $k$ and $a$ are approached. Otherwise, a logarithmic transformation allows linear least-squares fitting

$$F_A(k,a) = \sum_{i=1}^{N} \left( \ln\left[ \frac{q_0 t_i}{x_A(t_i)} - r_Y(t_i) y_R \right] - \ln[r_Z(t_i)] - a \ln(t_i) - \ln(k) \right)^2 \tag{54}$$

for $a$ and $\ln(k)$.

In the search for the field roughness, the friction-slope term, $S_f$, in the force-equilibrium equation

$$\frac{\partial y}{\partial x} = S_0 - S_f \tag{55}$$

can, with Manning roughness, be expressed in terms of the time-varying reference depth $y_R$ and the constant normal depth $y_N$, as

$$S_f = S_0 \left( \frac{y_N}{y_R} \right)^{10/3} \tag{56}$$

At the same time, the depth gradient $\partial y / \partial x$ can be approximated during advance by the expression

$$\frac{\partial y}{\partial x} = -\frac{w y_R}{x_A - x_R} \tag{57}$$

in which the correction term $w$ relates the depth gradient at $x_R$ to the average gradient.

Thus, the slope of the best-fit straight line through (0,0) and the series of points with the $-3/10$ term in Eq. (58) as abscissa and the left-hand side as ordinate, at times $t_n$ $(n=1,M)$ yields a current value of $y_N$

$$y(x_R,t_i) = y_N\left(1 + \frac{wy(x_R,t_i)}{S_0[x_A(t_i) - x_R]}\right)^{-3/10} \tag{58}$$

Valiantzas finds it both possible and unnecessary to update the $w$ correction factor, recommending simply a constant value of 0.3.

He reports convergence of the iterative scheme in three to six iterations. The method is reminiscent of Monserrat (1994) in its use of theoretical simulations to yield pertinent, time-varying shape factors $r_Y$ and $r_Z$ instead of assuming some constant values. The methods differ in that Monserrat utilizes a static database of prerun simulations, while Valiantzas' database is dynamic, run "on the fly" as required.

## Summary and Conclusions

Nearly two dozen methods proposed in the literature for estimating infiltration and roughness from field measurements of test irrigations have been reviewed. They differ in their data requirements, assumptions, ease of analysis, and accuracy. They have been divided into two broad categories. One features direct application of mass conservation expressed in terms of the infiltration parameters and then inverted in some way to extract them. The other is based on repeated simulation with a sequence of values of the field parameters, coupled to some kind of search procedure, an optimization, to minimize differences between simulation and measurement. The methods have been compared, especially in their theoretical foundations, common approaches, and differences. A new one-point technique has been proposed, along with suggestions for extending existing methods.

All methods assume that the inflow and outflow hydrographs are measured with reasonable accuracy. All methods for estimating roughness require that infiltration parameters are either known beforehand or estimated with one or another technique. Not all methods require Manning $n$, or its equivalent, to be either known or determined. Finally, the comments on the methods apply, by and large, to estimation from a single irrigation event.

Further testing is needed to determine the accuracy of these various methods under field conditions. Accuracy requirements depend upon the reason for the estimation and how the resulting parameter values will be put to use. Iterative or optimized solutions should be tested for reliability and uniqueness of convergence.

This library of techniques is intended as a source for further selection. From a practical standpoint, the user will be interested in minimizing the requirements for field data, while obtaining sufficient accuracy to satisfy evaluation, management, or design needs. More theoretically minded readers may choose to advance the capabilities of one or another method. The remaining four papers in this series prepared by the EWRI Task Committee on Soil and Crop Hydraulic Properties provide practical examples.

## Appendix I. Infiltration-Profile Shape Factor with Modified Kostiakov Intake and Power-Law Advance Functions

For a Kostiakov intake function augmented by a constant term and a final constant-rate term [Eq. (1)] and power-law advance, Eq. (35) become

$$A_Z = K(t-t_x)^a + B(t-t_x) + C$$

$$x_A = ft^h; \quad \frac{dx_A}{dt} = hft^{h-1}, \quad t \le t_L \tag{59}$$

$$x_A = L; \quad \frac{dx_A}{dt} = 0, \quad t > t_L$$

in order to include times both before and after completion of advance; $L$=length of run, and $t_L$=time to advance there. With the transformations and substitutions employed in developing Eq. (36), the infiltrated volume in Eq. (21) can be written for this general case as

$$V_Z = x_A[Kt^a r_{Z1} + Btr_{Z2} + Cr_{Z3}] \tag{60}$$

in which

$$r_{Z1} = \frac{h}{\lambda_U^h}\int_0^{\lambda_U}(1-\alpha)^a\alpha^{h-1}d\alpha \tag{61}$$

$$r_{Z2} = \frac{h}{\lambda_U^h}\int_0^{\lambda_U}(1-\alpha)\alpha^{h-1}d\alpha \tag{62}$$

$$r_{Z3} = \frac{h}{\lambda_U^h}\int_0^{\lambda_U}\alpha^{h-1}d\alpha \tag{63}$$

The factor $\lambda_U^h$ stems from the fact that in the postadvance period, $ft^h > L$, so, in order that $x_A$ appear explicitly in the infiltrated volume described by Eq. (60) ($x_A=L$, postadvance), $\lambda_U$ must be introduced. In general

$$\lambda_U = 1, \quad t \le t_L$$

$$\lambda_U = \frac{t_L}{t}, \quad t > t_L \tag{64}$$

The integral in Eq. (61) with $\lambda_U < 1$ is known as the incomplete $\beta$ function. During advance, with $\lambda_U=1$, it is simply the $\beta$ function. Multiplied by $h$, this is the shape factor $r_Z$ for Kostiakov infiltration. An approximation to the $\beta$ function was developed by Kiefer (1959), who expanded $(1-\alpha)^a$ in the binomial series, absolutely convergent for $0 \le \alpha \le 1$ and $a > 0$ (Courant 1937, Vol. 1, p. 406), and integrated the integral term by term to obtain the result for $r_Z$

$$r_Z = 1 - \frac{ah}{h+1} + \frac{a(a-1)h}{2\,!\,(h+2)} - \frac{a(a-1)(a-2)h}{3\,!\,(h+3)} + \ldots \tag{65}$$

The $\beta$ function is exactly expressible in terms of gamma functions (Abramowitz and Stegun 1964, formulas 6.2.1, 6.2.2), with the result

$$r_{Z1} = \frac{\Gamma(1+a)\Gamma(1+h)}{\Gamma(1+a+h)} \tag{66}$$

shown for orientation purposes as a family of curves in Fig. 8, with $a$ as the parameter. Eq. (66) is replaceable for all practical purposes by the closely fitting algebraic expression of Christiansen et al. (1966) [Eq. (37)]. Its 1% maximum error through the practical range of $a$ and $h$ is considerably better than the sum of four or five terms of Eq. (65).

During advance, it is clear that the growth of that part of the infiltrated volume stemming from the Kostiakov $k$ and $a$ is itself a power law, i.e.

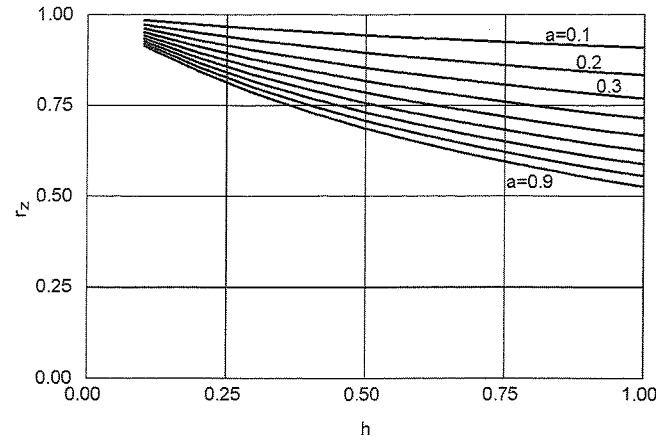


**Fig. 8.** Infiltration profile shape factor $r_Z$ during power-law advance with exponent $h$ and Kostiakov infiltration with exponent $a$

$$V_Z - x_A[Btr_{Z2} + Cr_Z] = x_A K t^a = g t^p \quad \text{with} \quad p = a + h \quad (67)$$

In the postadvance period, with $ft^h > L$, the corresponding series expansion (Scaloppi et al. 1995, corrected) taken to $N$ terms is

$$r_{Z1} = 1 + \sum_{i=1}^{N} \frac{(-\lambda_U)^i h}{h+i} \prod_{j=0}^{i-1} \frac{a-j}{i-j} \quad (68)$$

in which $\Pi$=product of $i$ terms. The terms of the series with $\lambda_U$ set to 1 are identical to those in Eq. (65). The more complex relationship, $r_Z(a,h,t/t_L)$, stemming from the incomplete beta function is plotted for comparison in Fig. 9, for one particular value of $a$=0.5. The curves are distinguished by the value of the parameter $h$. Values of $a$ smaller than 0.5 raise the values of $r_Z$, clustering them closer to 1.0, while larger $a$ values cause the curves to spread out (not shown).

The integrals in Eqs. (62) and (63) are easily evaluated, so that

$$r_{Z2} = \left(1 - \frac{h\lambda_U}{h+1}\right) \quad (69)$$

in the general case, or, during advance

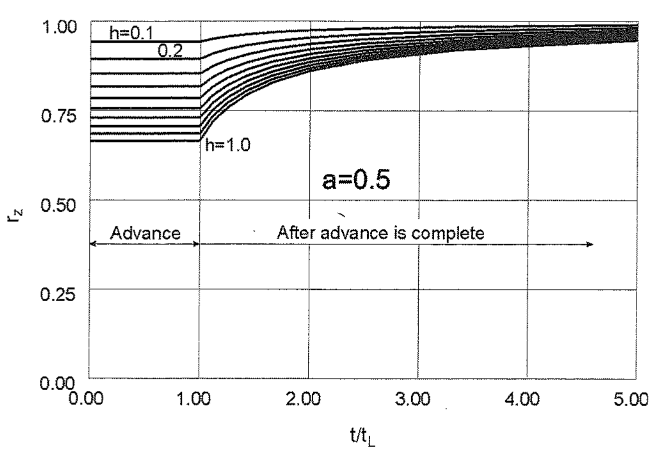


**Fig. 9.** Infiltration profile shape factor $r_Z$ during and after power-law advance with exponent $h$ and Kostiakov infiltration with exponent $a$

$$r_{Z2} = \left(\frac{1}{h+1}\right) \quad (70)$$

Finally

$$r_{Z3} = 1 \quad (71)$$

in all instances.

## Appendix II. Evaluation of Merriam and Clemmens (1985) Time-Rated Family from a Single Advance Point

As noted in Strelkoff and Clemmens (2001), an assumption that the field soil can be characterized by the Merriam and Clemmens (1985) time-rated families (similar to assuming an SCS family, as reported by NRCS/USDA (SCS) (1984), and as was done earlier by Valiantzas et al. (2001)] allows a determination of Kostiakov $k$ and $a$ from a single measurement of advance, say, to the end of the field. In the time-rated families, $k$ and $a$ are related through an empirical determination of a statistically significant relationship between $a$ and $t_{100}$, the time (in hours) to infiltrate a depth of 100 mm

$$a = 0.675 - 0.2125 \log_{10}(t_{100}) \quad (72)$$

With $k = 100/t_{100}^a$, $k(a)$ is given by

$$k = 10^{2-a[(0.675-a)/0.2125]} \quad (73)$$

For orientation purposes, Eqs. (72) and (73) are graphed in Figs. 10(a–c). It is clear that $k$ values below about 35 mm/hr$^a$ cannot be accommodated in the analyses. Fig. 10(b) shows also the relationship between $k$ and implied in the NRCS families for comparison. These empirical relationships were developed for the one-dimensional (vertical) infiltration in borders. To be used in furrows with any degree of success, $k$ and $K$ should be related through a physical wetted perimeter rather than furrow spacing. In the case of the NRCS families, that would be the empirical wetted perimeter [Eq. (20), Strelkoff et al. 2009], whereas with the time-rated families a reasonable choice would be the upstream wetted perimeter or wetted perimeter at normal depth.

Of the two values of $a$ evident in Fig. 10(b) for any $k$ value, only the larger is pertinent; the empirical function, Eq. (72), is valid only between about $a$=0.3 and $a$=0.8 (Merriam and Clemmens 1985).

A mass balance written for the entire inflow volume at the time $t_2$ is

$$G(a,h) = \overline{\overline{Q}}_0 t_2 - W_N k(a) t_2^a r_z(a,h) x_{A2} - \overline{A}_y x_{A2} = 0 \quad (74)$$

in which $G$=imbalance in volume (reduced to zero when the calculations have yielded the correct infiltration parameters); $\overline{\overline{Q}}_0$ =time-averaged inflow rate to the furrow or border strip, and $\overline{A}_Y$=distance-average cross-sectional area of the surface stream.

If in Eq. (74), the exponent $h$ in $r_Z$ could be shown dependent only on $\tilde{a}$, Eq. (74) could be solved for $a$ and Eq. (73) for $k$, thus completing the evaluation. Indeed, the first of Eq. (41) can be rewritten as

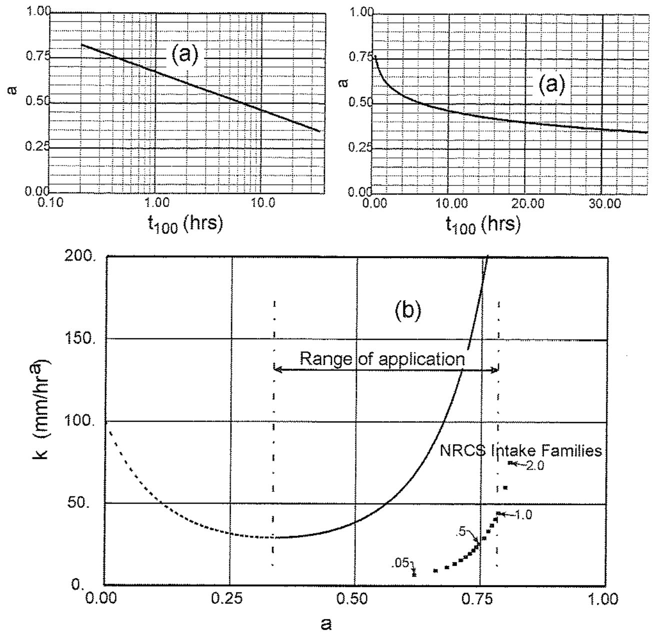


**Fig. 10.** (a) Empirical curves relating Kostiakov $a$ to $t_{100}$, the time to infiltrate 100 mm (Merriam and Clemmens 1985); (b) implied variation of $k$ with $a$. Values for $a<0.3$ are shown only for theoretical interest. Variation of $k$ with $a$ in NRCS intake families is shown for comparison.

$$h=\frac{\log\left(\frac{x_{A1}}{x_{A2}}\right)}{\log\left(\frac{t_1}{t_2}\right)}=\frac{\log r_x}{\log r_t} \tag{75}$$

The quantity, $r_t$, is arbitrary and user selected, typically, $r_t=1/2$, while $r_x$ can be shown a function of the unknown $a$ alone. Indeed, with $x_A=V_z/(r_z\cdot W_N\cdot t^a)$

$$r_x=\frac{x_{A1}}{x_{A2}}=\frac{V_{Z1}}{V_{Z2}r_t^a}=\frac{V_{Q1}-\overline{A}_Y r_x x_{A2}}{V_{Z2}r_t^a} \tag{76}$$

which can be solved for $r_x$ in terms of $a$ and known quantities

$$r_x(a)=\frac{V_{Q1}}{r_t^a V_{Z2}+\overline{A}_Y x_{A2}}=\frac{r_t\overline{\overline{Q}}_0 t_2}{r_t^a[\overline{\overline{Q}}_0 t_2-\overline{A}_Y x_{A2}]+\overline{A}_Y x_{A2}} \tag{77}$$

Eq. (74) reduces now to a single nonlinear equation in $a$. The left side of Eq. (74) is graphed in Fig. 11 for a sample set of physical

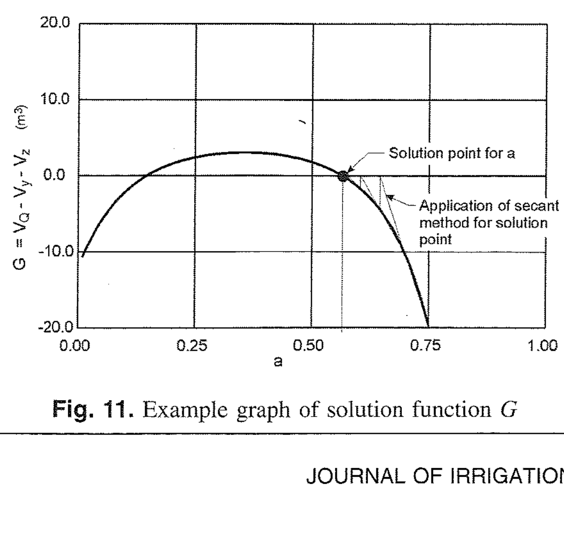


**Fig. 11.** Example graph of solution function $G$

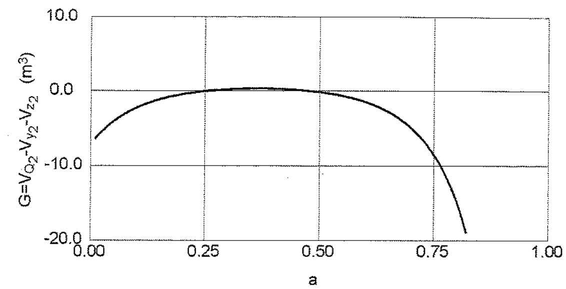


**Fig. 12.** Example of a poorly posed problem: small differences in measurements or assumptions would lead to large changes in the root, $a$, or no root

conditions; the larger root is the correct one. Iterative solution for $a$ is straightforward, provided the average cross-sectional area of the surface stream is not assumed so large that the function $G$ in Fig. 11 has no root. The secant method of solution (Press et al. 1992), starting with a first guess, $a=0.8$, and a second guess, $a=0.75$, to establish the slope of the initial secant, converges to the correct root ($>0.3$); however, as can be seen in Fig. 12, the physical parameters can be such as to lead to a poorly posed mathematical problem, with small changes in measured or assumed physical variables leading to large changes in the solution for $a$. As seen in Fig. 10, an overly large $a$ coincides with an overly large $k$ as well, leading to Kostiakov parameters, which consistently overestimate infiltration, a result of underestimated Manning $n$ or surface-stream shape factor, $r_Y$. It is also clear that measurement errors and different estimates of $V_Y$ can result in *no* possible solution for $a$.

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